OPTIMIZATION OF PRINCIPAL ROBIN EIGENVALUES ON 2-MANIFOLDS AND UNBOUNDED CONES

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In this talk, we will focus on optimization for the lowest eigenvalue of the Robin Laplacian with a negative boundary parameter on a compact, smooth, simply-connected, two-dimensional manifold with C^2-boundary of a fixed length. The main novelty compared to the better-understood Euclidean case is that the eigenvalue is optimized in the sub-class of manifolds, for which the Gauss curvature satisfies the pointwise inequality $K \leq K_\circ$ for a fixed $K_\circ \in \mathbb{R}$. This constraint on the curvature naturally enters into the problem. Our main result can be concisely formulated as follows: *the geodesic disk on the manifold of the constant Gauss curvature $K_\circ$ is a maximizer*.

Moreover, we will discuss a result on the optimization of the lowest Robin eigenvalue on an unbounded three-dimensional Euclidean cone Λ with a C^2-smooth, simply-connected cross-section $\Lambda \cap \mathbb{S}^2$ of a fixed perimeter. We prove that the cone with a circular cross-section is a maximizer. The argument relies on the same technique as for 2-manifolds, which is now applied slice-wise to the manifolds $\Lambda \cap (r\mathbb{S}^2)$ for each $r > 0$.

This talk is based on a joint work with Magda Khalile.