An introduction to elliptic corner problems via the example of polygonal metamaterials

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Metamaterials are characterized by negative constitutive coefficients in contrast with an exterior medium which has positive coefficients. We address operators written in variational form as

$$u \longmapsto \left(v \mapsto \int_\Omega A(x)\nabla u(x)\cdot\nabla v(x)\,dx \right), \quad u,v\in H^1(\Omega),$$

where Ω is the union of two subdomains Ω_a and Ω_b in which the coefficient A takes the two distinct values $a > 0$ and $b < 0$, respectively. The underlying operator $\mathscr{P}$ is bounded from $H^1(\Omega)$ into its dual space $H^1(\Omega)'$. We assume that Ω_a and Ω_b are polygonal in 2 dimensions. Natural questions arise (and are recently addressed in the literature):

(i) Is the operator $\mathscr{P}$ Fredholm from $H^1(\Omega)$ into $H^1(\Omega)'$?

(ii) Does $\mathscr{P}$ enjoy (limited) regularity shifts?

We name as "standard model" the elliptic theory developped after the works of Kondratev, Maz'ya and others. We intend to revisit this standard model, e.g. interior ellipticity, covering boundary conditions, and Mellin symbol at a corner, in order that it applies to the metamaterial case.